2. Review of Atmosphere Dynamic and Thermodynamic

Many formulations of the problems related to weather predictions have been studied heretofore in dynamic meteorology. Efforts in the hydrodynamic theory of weather prediction have resulted the series of quite complete and physically meaningful formulations of these problems, and at the present accept as the basis for the development of numerical methods. The formulations and methods of weather model problem will be continually improved by the accumulations of new information and technology about mechanism underlying atmospheric processes and computer sciences.

Based on atmospheric processes, numerical weather model has two basic directions being pursued in the theory of short-range weather predictions. The first attempted to improve the barotropic model by using balance equations; the other tried to create baroclinic model using the quasigeostropic approximations (Marchuk,1974). By evaluating the result of studies in the field of short-range weather prediction, it seems proper to conclude that the barotropic models lack a proper physical basis in that they only consider the redistribution of kinetic energy by wave dispersion processes. Baroclinic quasigeostropic models was admitted more complicated in transitions from internal energy to kinetic energy and vice versa. Barotropic predictions model only occasionally predictable, whereas the baroclinic quasigeostropic model could be describe up to 60-70% of all cases of weather phenomena. Short-range weather prediction has temporal and spatial resolution that could not describe for large area or weather conditions in the future. So, extended-range prediction is the most urgent scientific problems of weather prediction, since prognostic schemes for 3-5 days ahead must contain features from both short and long-range weather prediction.

Numerical weather model was developed based on quantitative theory of weather system; there arise various problems of formulation, both of the equations themselves and of the mathematical methods for their solution. The first problem is associated with the formulations of the basic equations for the dynamics and thermodynamics of atmosphere processes, which involve a variety of transitions between different forms of energy. This is primarily a physical problem. And the second problems are computational mathematic application and computer technology.

2.1. Atmosphere dynamic processes

Dynamic meteorology is the study of those motions of the atmosphere that are associated with weather and climate. For all such motions the discrete molecular nature of the atmosphere can be ignored, and the atmosphere can be regarded as a continuous fluid medium, or continuum (Holton, 1992). A point in the continuum is regarded as a volume element that is a very small compared with the atmosphere volume under consideration but still contains a large number of molecules. The expression air parcel and air particle are both commonly used to refer to such a point. The various physical quantities that characterize the state of the atmosphere are assumed have unique values at each point in the atmosphere continuum.

Dynamic process consider with the force that influences the motions of atmosphere. Based on the fundamental physical law of conservations of mass, momentum and energy, the forces that influence's can be classified as either body force or surface force. Developed from fundamental physical law, the forces that influences atmospheric process can be divided into two parts.

2.1.1. The fundamentals force

a. Pressure gradient force

Consider an infinitesimal volume element of air, $\delta V = \delta x\ \delta y\ \delta x$, centered at the point x_0, y_0, z_0 as illustrated in Figure 2.1. Below Figure can be expressed in a Taylor series expansion as :

$$p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2} + \text{higher} - \text{order expression}$$

$$F_{Ax} = -\left(p_0 + \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z$$

$$F_{Bx} = +\left(p_0 - \frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z$$

$$F_x = F_{Ax} + F_{Bx} = -\frac{\partial p}{\partial x}\delta x \delta y \delta z \qquad (2.1)$$

The mass *m* of the differential volume element is a simply the density ρ times the volume: $m = \rho\ \delta x \delta y \delta z$. Thus, the x, y and z component of pressure gradient force per unit mass is:

$$\frac{F_x}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$
$$\frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$
$$\frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

So the total pressure gradient force per unit mass is :

$$\frac{F}{m} = -\frac{1}{\rho}\nabla p$$

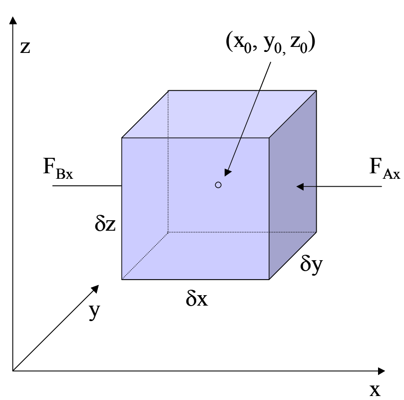

Figure 2.1. The x component of the pressure gradient force acting on a fluid element

b. Gravitational force

Newton's law of universal gravitation states that any two elements of mass in the universe attract each other with a force proportional to their masses and inversely proportional to the square of the distance separating them. The equation is:

$$F_g = -\frac{GMm}{r^2}\left(\frac{r}{r}\right)$$

Fg	:	Gravitational force
G	:	Gravitational constant
M and m	:	Mass of first and second elements
r	:	distance from center of M and m elements

Thus, if the earth is designated as mass M and m is a mass element of the atmosphere, the force per unit mass exerted on the atmosphere by the gravitational attraction of the earth is:

$$F_g \equiv g^* = -\frac{GM}{r^2}\left(\frac{r}{r}\right) \qquad (2.4)$$

In dynamic meteorology it is customary to use as a vertical coordinate the height above mean sea level, modification of the equation is :

$$g^* = \frac{g_0^*}{(1 + z/a)^2} \qquad (2.5)$$

Where :

$g_0^* = -(GM/a^2)(r/r)$ (2.2)

$r = a + z$

a : the mean of radius of the earth

z : the distance above mean sea level

note : For meteorology applications z<< a

c. Viscous force

For the atmosphere below 100 km, viscous force is so small that molecular viscosity is negligible except in a thin layer within few centimeters of the earth's surface where the vertical shear is very large (1992, Holton). So, for local phenomena, viscous force will be influences such as orography rainfall type. The vertical shearing stress on air can be seen below figure.

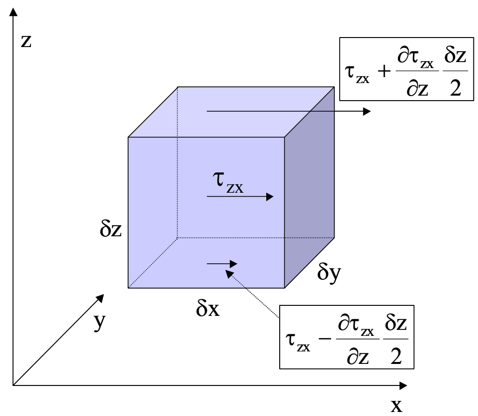

Figure 2.2. The x component of the vertical shearing stress on air component

If the mass of component is ρ δxδyδz, then the viscous force per unit mass owing to vertical shear of the component of motion in the x direction is :

$$\frac{1}{\rho}\frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) \quad (2.6)$$

Where :

μ : constant, derived from kinematics viscosity coefficient ($\nu=\mu/\rho$)

u : air velocity

The resulting frictional forces components (F_r)per unit mass in the three Cartesian coordinate directions are :

$$F_{rx} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (2.7a)$$

$$F_{ry} = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \quad (2.7b)$$

$$F_{rz} = \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \quad (2.7c)$$

2.1.2. Non inertial reference and "Apparent" force

a. Centrifugal force

The earth and air on above of surfaces are rotating system. Application of Newton's second law describes motion relative to this rotating coordinate system, include an additional apparent force, it is called centrifugal force, which just balances the force of the string. The centrifugal force is equivalent to the inertial reaction of the element on the atmosphere. It could be equal or opposite to the centripetal force. The equations was used to describe their force is centripetal acceleration such as :

$$\frac{dV}{dt} = -\omega^2 r \quad (2.8)$$

Where :

V : velocity

w : angular velocity

r : radius of the string

b. Gravity force

A particle of unit mass is at a rest on the surface of the earth. Observed in a referenced frame rotating with the earth, is subject to a centrifugal force $\Omega^2 R$, where Ω is the angular speed of rotation of the earth and R the position vector from the axis of rotation to the particle. The weight of a particle of mass m at a rest on the earth surface's, which is just the reactions force of the earth on the particle, will generally be less than the gravitational force mg* because the centrifugal force partly balances the gravitational force. The equations is :

$$g \equiv g + \Omega^2 R \quad (2.9)$$

Gravity can be represented in term of the gradient of a potential function Φ, called the geopotensial:

$$\nabla \Phi = -g \quad (2.10)$$

If the value of geopotensial is set to zero at mean sea level, the geopotensial Φ (z) at the height z is just the work required to raise a unit mass to height z from mean sea level.

$$\Phi = \int_0^z g dz \qquad (2.11)$$

c. Coriolis force

The mathematical form for the Coriolis force due to motion relative to the rotating earth can be obtained by considering the motion of hypothetical particle of unit mass that is free to move on a frictionless horizontal surface on the rotating earth. If the particle is initially at rest with respect to the earth, the only forces acting on it are the gravitational forces and apparent centrifugal force owing the rotation of the earth.

Letting Ω be magnitude of the angular velocity of the earth, R the position vector from the axis of rotation particle and u the eastward speed of the particle relative to ground, the total centrifugal force is:

$$\left(\Omega + \frac{u}{R}\right)^2 R = \Omega^2 R + \frac{2\Omega u R}{R} + \frac{u^2 R}{R^2} \qquad (2.12)$$

The first term on the right is just the centrifugal force owing the rotation of the earth. This is included in gravity. The other two terms represent deflecting forces, which act outward along the vector R. The $2\Omega u(R/R)$ is the Coriolis force owing to relative motion parallel to a latitude circle.

Coriolis force can be divided into component in the vertical and meriodional directions such as in Figure 2.3. Relative motion along the east-west coordinate produces accelerations in the north-south directions give by below equations:

$$\left(\frac{dv}{dt}\right)_{Co} = -2\Omega u \sin\Phi \qquad (2.13)$$

and the acceleration in the vertical direction :

$$\left(\frac{dv}{dt}\right)_{Co} = -2\Omega u \cos\Phi \qquad (2.14)$$

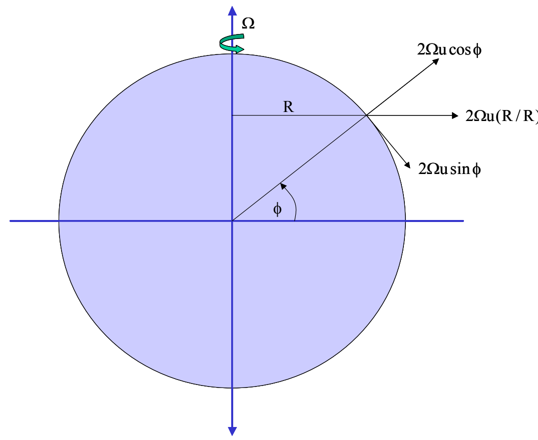

Figure 2.3. Component of the Coriolis force owing to relative motion along a latitude circle

Letting δR designate the change in the distance to the axis of rotation for a southward displacement from latitude ϕ_0 to latitude $\phi_0+\delta\phi$, so the conservation of angular momentum :

$$\Omega R = \left(\Omega + \frac{\delta u}{R + \delta R}\right)(R + \delta R)^2 \qquad (2.15)$$

If δu is the eastward relative velocity when the particle has reached latitude $\phi_0+\delta\phi$, so based on second order differential above equation:

$$\delta u = -2\Omega \delta R = +2\Omega a \delta\phi \sin\phi_0$$

where : $\delta R = -a\delta\phi \sin\phi_0$, a is the radius of the earth

Dividing through by the time increment δt and taking the limit as $\delta t \to 0$, obtain from the above equations:

$$\left(\frac{du}{dt}\right)_{Co} = 2\Omega a \frac{d\phi}{dt} \sin\phi_0 = 2\Omega v \sin\phi \qquad (2.17)$$

where : $v = a\, d\phi/dt$ is the northward velocity component

2.2. Atmosphere thermodynamic processes

Atmosphere thermodynamic process is the mechanism of the relations between atmosphere element such as temperature, pressure, and air density on the air column. Thermodynamic of the atmosphere consider about the equations of dry air, water vapor and moist air also their process. Processes that occur in the atmosphere with the relationship between the mechanical work done by the system and the heat the system receives, as expressed by the laws of Thermodynamic (Wallace and Hobbs, 1977). Describing about atmosphere process will be explained on path 2.1.3, 2.1.4 and 2.1.5 owing including dynamic and thermodynamic atmosphere relations.

2.2.1. The gas laws

Relational between temperature, air density and pressure can be described by an equation of state. All gases are found to follow approximately the same equation of state over a wide range conditions. This equation of state is referred to as the ideal gas equation. The ideal gas equation is:

$$pV = mRT \qquad (2.18)$$

where :
p : pressure
V : volume
M : mass
R : gas constant
T : Temperature

Derived from above equation, the density $\rho = m/V$, so the equation is:

$$p = \rho RT \qquad (2.19)$$

For a unit mass of gas, above equation could be write:

$$p\alpha = RT \qquad (2.20)$$

2.2.2. Virtual temperature

Consider a volume V of moist air at temperature T and total pressure p which contains mass m_d of dry air and mass m_v of water vapor. The density ρ of the moist air is given by:

$$\rho = \frac{m_d + m_v}{V} = \rho_d^{'} + \rho_v^{'} \qquad (2.21)$$

where ρ_d is the density which the same mass of dry air would have if it alone occupied the volume V and ρ_v the density which the same mass of water vapor would if it alone occupied the volume V. Applying the ideal gas equations to the water vapor and dry air also used Dalton's law of partial pressure will produces below equations:

$$\rho = \frac{p - e}{R_d T} + \frac{e}{R_v T} \qquad (2.22)$$

or,

$$\rho = \frac{p}{R_d T}\left(1 - \frac{e}{p}(1 - \varepsilon)\right)$$

where : $\varepsilon = R_d / R_v = M_w / M_d \equiv 0.622$

Above equation can be written as :

$$p = R_d \rho T_v$$

$$T_v = \frac{T}{1 - (e/p)(1 - \varepsilon)} \qquad (2.23)$$

T_v is virtual temperature. It has been appropriate temperature as considering the fact that mass of water molecules less than mass of dry air molecules (Pawitan, 1989). If this fictitious temperature, rather than the actual temperature, is used for moist air, the total pressure p and density r of the moist air are related by the ideal gas equation with the gas constant the same as that for a unit mass of dry air (R_d).

2.2.3. Specific heats

The specific heat of the material is the ratio of rate of quantity of heat (dq) and temperature (dT). It is defined, in this way can have any number of values depend on how the material changes while it receives the heat. If the volume of the material is kept constant, a specific heat at constant volume (c_v) is defined by:

$$c_v = \left(\frac{dq}{dT}\right)_{\alpha const}$$

Define by the specific volume (?), the internal energy (du) become equal with dq

$$c_v = \left(\frac{du}{dT}\right)_{\alpha const}$$

Based on the first law thermodynamic, both above equation can be written:

$$dq = c_v dT + pd\alpha \qquad (2.24)$$

If the pressure of the material is kept constant, a specific heat at constant volume (c_p) is defined by:

$$c_v = \left(\frac{dq}{dT}\right)_{pconst}$$

The quantity of heat at constant pressure can be written :

$$dq = c_v dT + d(p\alpha) - \alpha dp \qquad (2.25a)$$

$$dq = (c_v + R)dT - \alpha dp \qquad (2.25b)$$

So, specific heat at constant pressure can be determined by specific heat at constant volume.

$$c_p = c_v + R \qquad (2.26a)$$

$$dq = c_p dT - \alpha dp \qquad (2.26b)$$

2.2.4. Enthalpy

Discuss about it, consider on the process in atmosphere if heat is added to an air column or system at constant pressure, then the specific volume of the material increases from ?$_1$ to ?$_2$, the work done by unit mass of the material is $p(\alpha_2 - \alpha_1)$. Therefore, the heat added to a unit mass of the air column can be written (considering with internal energy).

$$dq = (u_2 - u_1) + p(\alpha_2 - \alpha_1)$$
$$= (u_2 + p\alpha_2) - (u_1 + p\alpha_1)$$

(2.27)

If h is the enthalpy of a unit mass and defined by:

$$h \equiv u + p\alpha$$

$$dh = du + d(p\alpha) \qquad (2.28)$$

So, quantity of heat can be written :

$$dq = dh - \alpha dp$$

$$dh = c_p dT$$

$$h = c_p T \qquad (2.29)$$

Enthalpy can be explained by developing of air column at constant pressure, so that the air parcel moving in a hydrostatic atmosphere neither gains nor loses heat.

2.3. Structure of the Static Atmosphere

Structure of the static atmosphere determine the relations and values of pressure, temperature and air density. State conditions that influence the relations and value can be derived from hydrostatic equations, geopotensial terminology, and hypsometric equations and implemented in the vertical coordinate.

2.3.1. The hydrostatic equations

The atmosphere is in motions at all times. In the absence of atmosphere motions the gravity force must be exactly balanced by the vertical component of the pressure gradient force (Holton, 1992). Consider a vertical column of air with unit cross-sectional area, the mass of air between heights z and z+ dz in the column is ρdz, where the ρ is air density at height z. The force acting on air column such as in Figure 2.4 due to the weight of the air is $g\rho dz$, where g is the accelerations due to gravity at height z. So, the balanced of forced in the vertical requires that :

$$\frac{dp}{dz} = -\rho g \qquad (2.30)$$

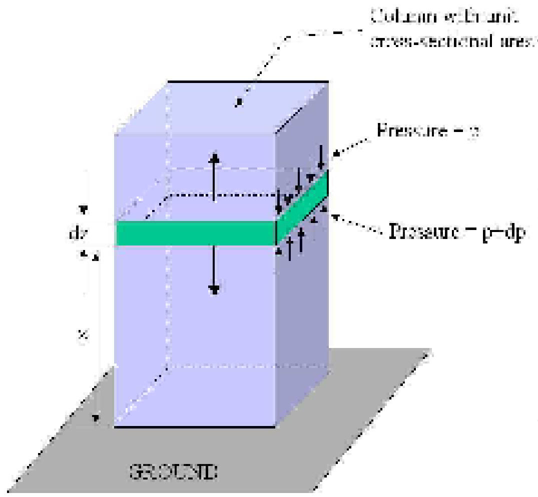

Figure 2.4. Balance of force for hydrostatic equilibrium (Holton, 1992)

Above equations is termed the hydrostatic equation. Hydrostatic balance condition provides an excellent approximation for the vertical dependence of the pressure field in the real atmosphere and intense small-scale system such as squall lines. Integrating from a height z to the top of the atmosphere can be written:

$$p(z) = \int_{z}^{\infty} \rho g dz \qquad (2.31)$$

That is, the pressure at level z is equal to the weight of the air in the vertical column of unit cross-sectional area lying above that level. If the mass of earth's atmosphere were uniformly distributed over the globe, the pressure at sea level would be 1013 mb, which is referred to as normal atmosphere pressure and abbreviate as 1 atm (Wallace and Hobbs, 1977)

2.3.2. Geopotential Height

According to Hobbs and Wallace (1977), the geopotential (Φ) is defined as the work that must be done against the earth's gravitational field in order to raise a mass of 1 kg from sea level to that point. In other word, geopotential (Φ) is the gravitational potential for unit mass. The basic of geopotential equations is:

$$d\phi = g.dz = -\alpha.dp \qquad (2.32)$$

The geopotential Φ (z) at height z is thus given by:

$$\phi(z) = \int_{0}^{z} g dz \qquad (2.33)$$

and the geopotential height (Z) is also defined by equations:

$$Z = \frac{\phi(z)}{g_0} = \frac{1}{g_0}\int_{0}^{z} g dz \qquad (2.34)$$

2.3.3. Scale height the hypsometric equations

In meteorological practice it is not convenient to deal with air density which can't be measured directly. By use the hydrostatic and geopotential equation, can be integrated pressure at levels p_1 and p_2 as differential vertical layers with geopotential Φ_1 and Φ_2. Integration of the geopotential equations in the vertical yields a form of the hypsometric equations.

$$\phi_2 - \phi_1 = -R_d \int_{p_1}^{p_2} T_v \frac{dp}{p} \qquad (2.35a)$$

$$Z_2 - Z_1 = \frac{R_d}{g_0} \int_{p_1}^{p_2} T_v \frac{dp}{p} \qquad (2.35b)$$

The temperature of the atmosphere generally varies with height. Thus, the variation of geopotential with respect to pressure depends only on temperature. In this case integrated a mean virtual temperature T_v with respect to ln p will result the equations that expressed the scale height.

$$Z_2 - Z_1 = \frac{R_d T_v}{g_0} \ln\left(\frac{p_1}{p_2}\right) = H \ln\left(\frac{p_1}{p_2}\right) \qquad (2.36)$$

where the scale height H is define as :

$$H = \frac{R_d T_v}{g_0} = 29.3 T_v \qquad (2.37)$$

2.3.4. Pressure as a vertical coordinate

Horizontal components of the pressure gradient force are evaluated by partial differentiation holding z constant. However, when pressure is used as the vertical coordinate horizontal partial derivatives must be evaluated holding p constant. Transformation of the horizontal pressure gradient force from height to pressure coordinate can be seen in Figure 2.5. Considering from x-z plane, from Figure 2. 5 can be define below equation.

$$\left[\frac{(p_0+\delta p)-p_0}{\delta x}\right]_z = \left[\frac{(p_0+\delta p)-p_0}{\delta z}\right]_x \left(\frac{\delta z}{\delta x}\right)_p$$

Above equations have the indicated subscripts that remain constant in evaluating the differentials. Consider on evaluating the pressure at the one layers, we can use limitation the z value into 0. Taking the limits $\delta x, \delta z \to 0$, will obtain:

$$\left(\frac{\partial p}{\partial x}\right)_z = -\left(\frac{\partial p}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_p$$

Consider with the hydrostatic equation:

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -g\left(\frac{\partial z}{\partial x}\right)_p = -\left(\frac{\partial \phi}{\partial x}\right)_p \quad (2.38)$$

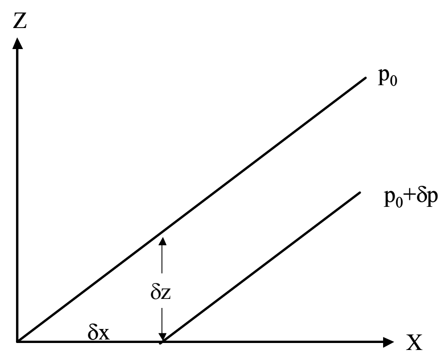

Figure 2.5. Slope of pressure surfaces in the x,z plane (Holton, 1992)

In many numerical weather prediction models pressure normalized by the pressure at the ground $[\sigma \equiv p(x,y,z,t)/p_s(x,y,t)]$ is used as a vertical coordinate (Holton, 1992). This choice guarantees that the ground is a coordinate surface ($\sigma \equiv 1$) even in the presence of spatial and temporal surface pressure variations.

General expression for the horizontal pressure gradient, which is applicable to any vertical coordinate s=s(x,y,z,t) that is a singled valued monotonic functions of height. In the Figure 2.6 present the horizontal distance x the pressure difference evaluated along the surface s= constant, is related to that evaluated at z = constant. The equation is:

$$\frac{p_c - p_a}{\delta x} = \frac{p_c - p_b}{\delta z}\frac{\delta z}{\delta x} + \frac{p_b - p_a}{\delta x}$$

Taking the limits $\delta x, \delta z \to 0$, will obtain:

$$\left(\frac{\partial p}{\partial x}\right)_x = \frac{\partial p}{\partial z}\left(\frac{\partial z}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_z$$

Using identity $\partial p/\partial z \equiv (\partial s/\partial z)(\partial p/\partial s)$, above equations can be express :

$$\left(\frac{\partial p}{\partial x}\right)_x = \left(\frac{\partial p}{\partial x}\right)_z + \frac{\partial s}{\partial z}\left(\frac{\partial z}{\partial x}\right)_s\left(\frac{\partial p}{\partial s}\right) \quad (2.39)$$

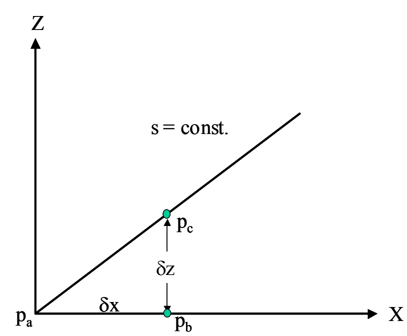

Figure 2.6. Transformation of the pressure gradient force to s coordinates (Holton, 1992)

2.4. Atmospheric Processes

Atmospheric process can be defined as change of physical atmospheric condition that expressed by weather phenomena. The physical process consider about hydro-thermodynamic atmosphere including many factors that influences it. Like as application of fundamental physic, in this model, weather phenomena can be simplified to explain a complex system.

2.4.1. Concept of an air parcel

In the atmosphere mechanisms problem, mixing is viewed as result of the random motions of individual molecules. Their mixing is important only within centimeter of the earth's surface and levels above the turbo pause (≈ 105 km). At intermediate levels, virtually all the vertical mixing is accomplished by the exchange of well-defined air parcels with horizontal dimensions ranging from a few centimeters to scale of the earth itself.

According to Hobbs and Wallace (1977), the behavior of air parcel of infinitesimal dimensions that assumed to be:

- Thermally insulated from its environment so that its temperature changes adiabatically as it rises or sinks
- Always at extract the same pressure as the environmental air at the same level, which is assumed to be in hydrostatic equilibrium
- Moving slowly enough that its kinetic energy is a negligible fractions of its total energy

2.4.2. The adiabatic lapse rate

Material changes its physical state without any heat being either added to it or withdrawn from it, the changing called adiabatic. Different with isotherm process, compressing volume a system that change by adiabatic process will be resulted the higher pressure than isotherm process. It owes that during the adiabatic compression the internal energy increases and therefore the temperature of the material rises. However, for the isothermal compression the temperature remains constant. Hence, $T_C > T_B$ and therefore $P_C > P_B$. Illustration of compression process can be seen in Figure 2.7.

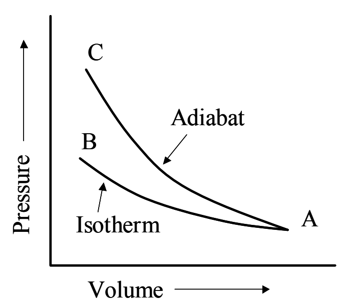

Figure 2.7. Representation of an isothermal (AB) and an Adiabatic (AC) transformations on a p-V diagram

Consider on temperature changing with adiabatic process, can be derived an expression for the rate of change of temperature with height of a dry air parcel which moves about in the earth's atmosphere while always satisfying the conditions listed. Using the adiabatic process can be derived that changing of heat is zero (dq=0), and the atmosphere is a hydrostatic equilibrium, so the equation are:

$$d(c_p T + \phi) = 0$$

$$-\left(\frac{dT}{dz}\right)_{dry} = \frac{g}{c_p} \equiv \Gamma_d \qquad (2.40)$$

Where Γ_d is called dry adiabatic lapse rate. If an air parcel expands as it rises in the atmosphere, its temperature will decrease with height.

When the atmosphere expanded or compressed adiabatically from its existing pressure and temperature to a standard pressure, it will have the potential temperature. To calculate potential temperature (θ), can be derived from pressure (p), temperature (T), the standard pressure (P_0) and adiabatic transformations.

$$c_p dT - \alpha dp = 0$$

$$\frac{c_p}{R}\frac{dT}{T} - \frac{dp}{p} = 0 \qquad (2.41)$$

Integrating above equation from p_0 and $T=\theta$,

$$\frac{c_p}{R}\int_{\theta}^{T}\frac{dT}{T} = \int_{p_0}^{p}\frac{dp}{p} \qquad (2.42)$$

$$\frac{c_p}{R}\ln\frac{T}{\theta} = \ln\frac{p}{p_0}$$

$$\left(\frac{T}{\theta}\right)^{c_p/R} = \frac{p}{p_0}$$

$$\theta = T\left(\frac{p_0}{p}\right)^{R/c_p}$$, for dry air R/c_p=0.286

(2.43)

The last equation is called Poison's equation.

2.4.3. Water vapor in the air

Weather prediction for tropical area should consider about water vapor, because there have higher humidity than other area. Understanding about water vapor be important to know including the process that accomplished. So far, to indicated the presence of water vapor in the air through the vapor pressure (e) which its exerts and its effect on the density of air by introducing the concept of a virtual temperature.

a. Moistures parameters

Mixing ratio (w)

The amount of water vapor in a certain volume of air defined as the ratio of the mass (m_v) of water vapor to the mass (m_d) of dry air.

$$w \equiv \frac{m_v}{m_d}$$

$$e = \frac{m_v/M_w}{m_d/M_d + m_v/M_w} p$$

$$e = \frac{w}{w+\varepsilon} p, \quad \text{where } \varepsilon = 0.622 \qquad (2.44)$$

Saturation vapor pressure

It is saturated with respect to a plane surface of pure water or ice surface. This condition occurs owing the air, which is an equilibrium state with respect to a plane surface. The magnitudes of the saturation vapor pressure depend only on temperature and they both increase rapidly with increasing temperature.

Saturation mixing ratio (w_s)

The saturation-mixing ratio with respect to water is defined as the ratio of the mass (m_{vs}) of water vapor in a given volume of air saturated with respect to a plane surface of water to the mass (m_d) of the dry air.

$$w_s = \frac{m_{vs}}{m_d}$$

$$w_s = \frac{\rho'_{vs}}{\rho'_d} = \frac{e_s/(R_v T)}{(p-e_s)/(R_d T)}$$

$$w_s = 0.622 \frac{e_s}{p - e_s}$$

$$w_s \approx 0.622 \frac{e_s}{p}, \; p >> e_s \qquad (2.45)$$

Relative humidity (RH)

Relative humidity is the ratio of the actual mixing ratio (w) to the saturation-mixing ratio (w_s) with respect to water at the same temperature and pressure

$$RH \equiv 100 \frac{w}{w_s} \qquad (2.46)$$

Dew point (Td)

The air temperature must be cooled at constant pressure in order to saturate with respect to a plane surface of water. According to Hobbs and Wallace (1977), dew point is the temperature at which saturations mixing ratio (w_s) with respect to water becomes equal to the actual mixing ratio (w).

$$RH \equiv 100 \frac{w_s(\text{at temperature Td and pressure } p)}{w_s(\text{at temperature T and pressure } p)}$$

Frost point

The air temperature must be cooled at constant pressure in order to saturate with respect to a plane surface of ice. It is define analogous ways to the corresponding definitions with respect to water.

Lifting condensation level

Rising unsaturated air expands and cools. If it rises high enough and has adequate moisture, it will be cool to its dew-point temperature and be saturated. The occurred level is referred to as the lifting condensation level, or LCL. If the air continues to rise, we would expect to see condensation with a cloud forming. The base of the cloud would be at the LCL and the vertical extent of the cloud would be determined by many factors, including the temperature and moisture content of the air above the LCL (Figure 2.8). According to NCAR (2001), the Lifting Condensation Level (LCL) is the level where condensation (saturation) occurs if one lifts an unsaturated surface parcel dry-adiabatically. Graphically on the skew-T plot it is the point where the dry adiabat (originating at the parcel temperature) and mixing ratio lines (originating at the parcel dew point temperature) intersect.

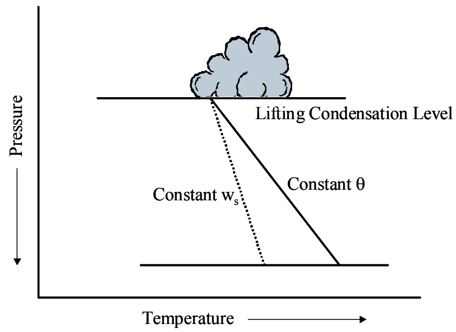

Figure 2.8. Location of lifting condensation level of a parcel of air at pressure p and temperature T and dew point Td on a pseudoadiabatic chart (Hobbs and Wallace, 1977)

2.4.4. Saturated-adiabatic and pseudoadiabatic processes

When an air parcel rises in the atmosphere, its temperature decrease with altitude at the dry adiabatic lapse rate until the air becomes saturated with water vapor. So far, the air parcel gets to the lifting result in the condensation of liquid water or may be deposition of ice which release latent heat. This fact will give consequent that the rate of decrease of temperature of the rising parcel become less. If all of the condensations product remain in the rising parcel, the process that occur still consider adiabatic and its can be called saturated adiabatic-process. In the other hand, if all the condensations products immediately fall out of the parcel of air, the process cannot strictly adiabatic; it's called pseudoadiabatic process.

To derive the rate of change in temperature with height of air parcel undergoing a saturated adiabatic process could be used below equations:

$$dq = c_p dT + gdz$$

$$-Ldw_s = c_p dT + gdz$$

$$\frac{dT}{dz} = -\frac{L}{c_p}\frac{dw_s}{dz} - \frac{g}{c_p}$$

$$\frac{dT}{dz} = -\frac{L}{c_p}\frac{dw_s}{dT}\frac{dT}{dz} - \frac{g}{c_p}$$

$$\Gamma_s \equiv \frac{dT}{dz} = \frac{\Gamma_d}{1+\left(\frac{L}{c_p}\right)\left(\frac{dw_s}{dT}\right)} \qquad (2.47)$$

where :

Γ_d : dry adiabatic lapse rate

$-L\,dw_s$: a unit mass of dry air due to condensations of liquid water

L : Latent heat

Γ_s : saturated adiabatic lapse rate

According to Hobbs and Wallace (1977), the magnitude of Γ_s is not constant, depends on the pressure and temperature. Since dws/dT is always positive, it follows from that $\Gamma_s < \Gamma_d$. Actual values of Γ_s range from about 4° near the ground in warm.

Such as description about pseudoadiabatic process, it is irreversible condensations process. When an air parcel is lifted above lifting condensation level so that condensations occurs and the product of its fall out, the latent heat gained by the air during this process will be retained if the air parcel returns to its original level. This fact will give some effect such as:

- Net increases in the temperature and potential temperature of the air parcel
- A decrease in moisture content. It is indicated by changes of mixing ratio, RH, dew point)
- No change in the equivalent potential temperature or wet bulb potential temperature.

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